Even-odd identities

As you may know, there are specific kinds of functions that are even functions, and specific kinds of functions that are odd functions.

If a function is even, it means you can replace x everywhere in the function with $-x$, and the function doesn't change. $f(x) = x^2$ would be an example, because you can replace x with $-x$ and get $f(x) = (-x)^2$ and after simplifying, you end up with $f(x) = x^2$ again, so the function didn't change.

A function is odd if you can replace x everywhere with $-x$ and end up with the original function multiplied by -1. $f(x) = x^3$ is an example, because you can replace x with $-x$ and get $f(x) = (-x)^3$ and after simplifying, you end up with $f(x) = -x^3$, which is just the original function multiplied by -1.

Even functions:	$f(-x) = f(x)$
Odd functions:	$f(-x) = -f(x)$

Even-odd trig identities are built from the fact that cosine is an even function and sine is an odd function. In other words, the even-odd identities are

$$\cos(-\theta) = \cos\theta$$

$$\sin(-\theta) = -\sin\theta$$

Let's look at how we can use even-odd identities to find values along the unit circle.

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Example

Find the values of $\cos(-\pi/3)$ and $\sin(-\pi/3)$.

Since the cosine function is even,

$$\cos\left(-\frac{\pi}{3}\right) = \cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$$

Since the sine function is odd,

$$\sin\left(-\frac{\pi}{3}\right) = -\sin\left(\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2}$$

Let's look at another example.

Example

Find the values of $\cos(-27\pi/4)$ and $\sin(-27\pi/4)$.

Since the cosine function is even,

$$\cos\left(-\frac{27\pi}{4}\right) = \cos\left(\frac{27\pi}{4}\right)$$

To find the angle that's coterminal to $27\pi/4$ in the interval $[0,2\pi]$, we'll take $27/4 = 6.75$. So we could rewrite $27\pi/4$ as

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$$6\pi + \frac{3\pi}{4}$$

Therefore, $27\pi/4$ is coterminal with $3\pi/4$, so

$$\cos\left(\frac{27\pi}{4}\right) = \cos\left(\frac{3\pi}{4}\right) = -\frac{\sqrt{2}}{2}$$

Since the sine function is odd,

$$\sin\left(-\frac{27\pi}{4}\right) = -\sin\left(\frac{27\pi}{4}\right)$$

We know that

$$-\sin\left(\frac{27\pi}{4}\right) = -\sin\left(\frac{3\pi}{4}\right) = -\frac{\sqrt{2}}{2}$$

We can use the even-odd identities for cosine and sine to determine how $\tan(-\theta)$, $\sec(-\theta)$, $\csc(-\theta)$, and $\cot(-\theta)$ are related to $\tan\theta$, $\sec\theta$, $\csc\theta$, and $\cot\theta$, respectively:

$$\tan(-\theta) = \frac{\sin(-\theta)}{\cos(-\theta)} = \frac{-\sin\theta}{\cos\theta} = -\frac{\sin\theta}{\cos\theta} = -\tan\theta$$

$$\sec(-\theta) = \frac{1}{\cos(-\theta)} = \frac{1}{\cos\theta} = \sec\theta$$

$$\csc(-\theta) = \frac{1}{\sin(-\theta)} = \frac{1}{-\sin\theta} = -\frac{1}{\sin\theta} = -\csc\theta$$

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$$\cot(-\theta) = \frac{\cos(-\theta)}{\sin(-\theta)} = \frac{\cos\theta}{-\sin\theta} = -\frac{\cos\theta}{\sin\theta} = -\cot\theta$$

Therefore, we have four additional even-odd identities, and we can group all the even-odd identities together as

Even identities

$$\cos(-\theta) = \cos\theta$$

$$\sec(-\theta) = \sec\theta$$

Odd identities

$$\sin(-\theta) = -\sin\theta$$

$$\csc(-\theta) = -\csc\theta$$

$$\tan(-\theta) = -\tan\theta$$

$$\cot(-\theta) = -\cot\theta$$

Example

Find the values of all six trig functions at $-750°$.

If we take out two full $360°$ rotations from $750°$, you're left with $30°$, which means $-750°$ is coterminal with $-30°$.

$$750° = 720° + 30° = 2(360°) + 30°$$

The cosine and secant functions are even, so $-30°$ is the same as $30°$, and we can say

$$\cos(-750°) = \cos(-30°) = \cos(30°) = \frac{\sqrt{3}}{2}$$

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$$\sec(-750°) = \sec(-30°) = \sec(30°) = \frac{2}{\sqrt{3}}$$

For the secant function, we'll rationalize the denominator to get

$$\sec(-750°) = \sec(-30°) = \sec(30°) = \frac{2\sqrt{3}}{3}$$

The sine, cosecant, tangent and cotangent functions are odd, so we can say

$$\sin(-750°) = \sin(-30°) = -\sin(30°) = -\frac{1}{2}$$

$$\csc(-750°) = \csc(-30°) = -\csc(30°) = -2$$

$$\tan(-750°) = \tan(-30°) = -\tan(30°) = -\frac{\sin(30°)}{\cos(30°)} = -\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = -\frac{1}{2}\left(\frac{2}{\sqrt{3}}\right) = -\frac{\sqrt{3}}{3}$$

$$\cot(-750°) = \cot(-30°) = -\cot(30°) = -\frac{\cos(30°)}{\sin(30°)} = -\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = -\frac{\sqrt{3}}{2}\left(\frac{2}{1}\right) = -\sqrt{3}$$

Remember also that you really only need to find three of the trig functions directly, and then you can find the other three using reciprocal identities. You can find cosecant as the reciprocal of sine or vice versa, you can find secant as the reciprocal of cosine or vice versa, and you can find cotangent as the reciprocal of tangent or vice versa.

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